Parameter Identification of Aeroelastic Modes of Rotary Wings from Transient Time Histories

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This paper describes a new, improved, moving-block technique for the parameter identification of aeroelastic modes. The technique uses a least-squares solver in the time domain for the estimate of the dampings and the eigenvector components. Application of this method to analytical test cases shows that better accuracy is obtained as compared to the classical moving-block technique. Finally, results from a helicopter simulation are analyzed using this new approach.

Nomenclature

A = matrix of influence coefficients
A_k = amplitude coefficient
a_k = cosine coefficient
b_k = sine coefficient
C = damping matrix
C_{kp} = mean value of amplitude coefficient
c_k = complex constant
F_k = Floquet amplitude function
f_0 = rotor frequency
f_k = frequency of mode k
K = stiffness matrix
K_B = moving-block step size
l = number of blocks
M = mass matrix
M = total number of sampled data
m = number of modes
N_i = number of data in block of length T_i
T_i = period of mode i
T_M = maximum period of all modes
t = time
W_k = Floquet eigenvector function
X = vector of aeroelastic dof
x_n = discrete value of transient response
$\bar{x}_k$ = discrete Fourier transform coefficient
y = logarithm of Fourier transform amplitude
y_k = mean value of cosine coefficient
y_{k+m} = mean value of sine coefficient
α_k = phase estimate from cosine coefficient
β_k = phase estimate from sine coefficient
λ_k = complex eigenvalue
γ = phase of Fourier expansion
σ_k = damping of mode k
Δf = frequency resolution
Δt = time increment
Φ = phase angle
ϕ_k = phase of mode k
τ = time
ϵ = amplitude of Fourier expansion
μ = advance ratio

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Introduction

THE parameter identification of aeroelastic modes has been mainly considered in the analysis of experimental data. The frequency and damping of the active modes are estimated for various flight conditions and rotor speeds in order to establish the safe operating range for a given helicopter. References 1–3 are typical examples of such applications. One widely used method for the analysis of transient time histories is the moving-block technique.[4] Alternate methods such as the recursive least-squares (RLS) or recursive prediction error method (RPEM) use input/output time histories to identify transfer function parameters. The frequency and damping of the aeroelastic modes are obtained from the roots of the characteristic equation.[5] Another approach, known as the sparse time domain algorithm (STD),[6] reduces modal identification to a standard eigenvalue problem. This approach is attractive if the main interest is in the computation of the mode shapes. However, the large number of time data, measured at several locations, needed to run this method makes it costly compared to other methods if the main interest pertains to the frequency and damping identification. Also, postprocessing of the eigenvectors may be necessary to separate physical modes from computational ones. A different approach to modal identification[7] estimates model parameters (mass, stiffness, and damping matrices) from time response data. The frequencies, dampings, and eigenvectors are subsequently calculated using an eigensolver. These are some of the most used modal identification methods applicable to rotary wings and are a few of the large number of published methods in the more general subject of parameter identification.

The aim of this paper is to develop a fast and reliable method capable of identifying aeroelastic modes for both constant and periodic coefficients systems with no need for an initial guess of system order or parameters, or for separation of physical and computational modes. The moving-block technique is the only method that can be extended to solve for the periodic case and which satisfies the conditions just formulated. The only disadvantage is reliability, which is the subject of the following development.

Several helicopter simulations compute the transient aeroelastic response from nonlinear differential equations. The new challenge in aeroelastic application is to extract useful information from such results. The advantage of simulation results as compared to experimental results is the absence of the noise contamination problem. "Cleaner" data do not require filtering and are therefore easier to analyze. Nonlinear simulations give solutions to the following system of differential equations

$$M(t)\ddot{X} + C(t)\dot{X} + K(t)X = F(\dot{X}, X, t) \tag{1}$$

where M, C, and K are periodic with respect to rotor speed.

The preceding system is integrated in the time domain with the right-hand side set to zero.[8] The dynamic response results will be identified with the formal Floquet solution written below:

$$X(t) = \sum c_k W_k(t) e^{\lambda_k t} \quad (2)$$

where W_k is periodic with respect to rotor speed.

One of the most important features of rotary wing dynamics is the periodicity introduced by the rotor speed, although for a rotary wing flying at low to moderate forward speeds, the constant coefficient approximation can be applied to the system of differential equations of Eq. (1).

The time history of a selected degree of freedom, whether measured or computed, contains important modal information. For aeroelastic application, the dynamic characteristic of the dominant modes are crucial to know in order to determine the stability of a rotary wing. The method of choice to estimate the frequency and damping of the dominant modes from a given time history is the moving-block technique. However, the moving-block technique has limitations in the case of modes with close frequencies and when several aeroelastic modes are active. Also, the eigenvectors are not estimated. In what follows, a new method is proposed with none of the aforementioned limitations. This new method identifies the dominant frequencies in the frequency domain and the dampings and eigenvector components in the time domain. The validation of the proposed approach then follows for a number of selected examples. In particular, it will be shown this new approach can be extended to solve for the case of periodic eigenvectors.

Analysis

Development of the Method

The present approach describes a refinement of the well-known moving-block technique. A brief description of the moving-block technique is presented first. Consider the discretized time history of a given aeroelastic degree of freedom with time increment Δt:

$$[x_0 x_1 x_2 \cdots x_n \cdots x_M] \quad (3)$$

1) The first step is to perform a fast Fourier transform (FFT) to find the frequency content of the time signal. The frequency resolution

$$\Delta f = \frac{1}{N\Delta t}, \qquad N \leq M \quad (4)$$

can be adjusted to give better accuracy of the frequency spectrum.

$$\bar{x}_k = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi nk}{N}} \qquad k = 0,1,\ldots, N-1 \quad (5)$$

2) Once the complex Fourier coefficients $\{\bar{x}_k\}$ $k = 0,\ldots, N-1$ are computed, the second step is to identify the dominant frequencies. A visual display of the magnitudes of the complex Fourier coefficients

$$\{|\bar{x}_k|\} \qquad k = 0,\ldots, (N-1)/2 \quad (6)$$

is useful to pick the dominant frequencies. Without a display, the identification process can be difficult if the time signal is contaminated with noise. In case of significant noise, some appropriate filtering of the data needs to be performed to facilitate the search for the dominant frequencies. Note that it is sufficient to consider the Fourier amplitudes up to $(N-1)/2$ only because the symmetry of the FFT with respect to $N/2$. In case of a smooth Fourier spectrum, a local maximum will occur if the following condition is satisfied

$$|\bar{x}_{k_i-1}| < |\bar{x}_{k_i}| > |\bar{x}_{k_i+1}| \quad (7)$$

the corresponding dominant frequencies are then

$$f_i = k_i \Delta f \qquad i = 1,2,3,\ldots \quad (8)$$

3) The moving-block technique is founded on the following: assume a continuous time signal of the special form written as

$$x(t) = Ae^{-\sigma t} \sin(2\pi f t + \varphi) \quad (9)$$

It can be shown that, for subcritical damping, the application of the Fourier transform

$$\bar{x}(f) = \int_{\tau}^{\tau+T} x(t) e^{-j2\pi f t}\, dt \quad (10)$$

leads to the following important result.[4]

$$\ln|\bar{x}(f)| = -\sigma\tau + (\sigma/4\pi f)\sin 2(2\pi f\tau + \Phi) + C^{te} \quad (11)$$

If the FFT is performed on successive blocks of size $1/f_i$, the damping can be computed using the preceding result as shown next.

The total number of sampled data corresponding to dominant frequency f_i is then

$$N_i = 1/f_i \Delta t \quad (12)$$

A moving-block of length N_i is moved along the time axis with step size K_B; the Fourier amplitude is then

$$y_p = \ln|\bar{x}_{k_i}| \quad \text{for} \quad \{x_n\} \qquad n = p, p + N_i - 1 \quad (13)$$

$$\tau_p = (p-1)K_B \Delta t \qquad p = 1,2,\ldots, l \quad (14)$$

A plot of y vs τ can be constructed from which the slope $(-\sigma_i)$ is estimated using a linear least-squares fit to the curve $y(\tau)$

$$-\sigma_i = (l\Sigma\tau y - \Sigma\tau\Sigma y)/[l\Sigma\tau^2 - (\Sigma\tau)^2] \quad (15)$$

4) The moving-block technique yields good results for a small number of modes especially when the frequencies are well separated. In the case of a large number of modes and closer frequencies, the estimate of the dampings are greatly affected by frequency interaction or leakage. The presence of leakage sometimes leads to inaccurate results for the dampings. An alternate and more powerful method uses regression analysis on a moving block instead of the more common FFT for the estimate of the dampings. The general form of the transient time signal is written as

$$x(t) = \sum_{k=1}^{m} e^{-\sigma_k t}[a_k \cos(2\pi f_k t) + b_k \sin(2\pi f_k t)] \quad (16)$$

Steps 1 and 2 are used to find the frequencies

$$\{f_k\} \qquad k = 1,2,3,\ldots \quad (17)$$

from which a block size is selected as follows:

$$N_i = (1/f_i \Delta t) \quad (18)$$

The general form [Eq. (16)] of the transient time signal is specialized in the discrete time domain for each subsequent block as follows:

$$x_n = \sum_{k=1}^{m} [y_k \cos(2\pi f_k t_n) + y_{k+m} \sin(2\pi f_k t_n)] \quad (19)$$

the coefficients $\{y_k, y_{k+m}\}$ are estimated using the least-squares method. For the first block, $p = 1 (N = N_i)$, define

$$Y^T = [y_1 \cdots y_m y_{m+1} \cdots y_{2m}] \quad (20)$$

$$X^T = [x_0 \cdots x_{N-1}] \quad (21)$$

$$A = \begin{bmatrix} \cos 2\pi f_1 t_0 & \cdots & \cos 2\pi f_m t_0 & \sin 2\pi f_1 t_0 & \cdots & \sin 2\pi f_m t_0 \\ \cos 2\pi f_1 t_1 & \cdots & \cos 2\pi f_m t_1 & \sin 2\pi f_1 t_1 & \cdots & \sin 2\pi f_m t_1 \\ \cos 2\pi f_1 t_{N-1} & \cdots & \cos 2\pi f_m t_{N-1} & \sin 2\pi f_1 t_{N-1} & \cdots & \sin 2\pi f_m t_{N-1} \end{bmatrix} \tag{22}$$

The least-squares solution to the following problem is well known and is given below:

$$\min |X - AY|^2 \tag{23}$$

$$Y = (A^T A)^{-1} A^T X \tag{24}$$

The same process is repeated for block number 2,3,..., l. The amplitudes corresponding to frequency k and block number p are then

$$C_{kp} = \sqrt{y_k^2 + y_{k+m}^2} \begin{cases} k = 1, m \\ p = 1,2,3,\ldots, l \end{cases} \tag{25}$$

The following approximation

$$e^{-\sigma_{kp}(\tau_{p+1} - \tau_p)} \approx (C_{kp+1})/C_{kp} \tag{26}$$

for two consecutive blocks p and $p + 1$ yields

$$\sigma_{kp} \approx \frac{\ln(C_{kp}) - \ln(C_{kp+1})}{K_B \Delta t} \tag{27}$$

An estimate of damping for number k can be computed as an average value of the subsequent block estimates

$$\sigma_k \approx \Sigma \sigma_{kp}/(l - 1) \tag{28}$$

Damping Estimation

There are m different damping coefficients for every one of the following block sizes $N_1, N_2, \ldots, N_m$. The following weighted average for the global estimate of the damping coefficients is recommended. The larger sizes are considered to give better estimates because they contain more data

$$(\sigma_k)^{\text{weighted}}_{\text{average}} = (\Sigma N_i \sigma_k^i / \Sigma N_i) \tag{29}$$

with σ_k^i for mode k and block length N_i.

Estimation of the Amplitude and Phase of Eigenvectors

The relevance of eigenvector values for the study of the destablizing effect in aeroelasticity is discussed in Ref. 9.

The block with the largest number of points is considered because it gives better accuracy in least-squares estimation. Also, the first block $p = 1$ ought to be considered in order to retain the fast decaying modes at an acceptable amplitude level.

$$C_k = \frac{1}{T_M} \int_0^{T_M} A_k e^{-\sigma_k t}\, dt \tag{30}$$

$$A_k = \sqrt{a_k^2 + b_k^2} \tag{31}$$

$$T_M = \max\{T_i\} \qquad i = 1, m \tag{32}$$

Thus

$$A_k = C_k \frac{\sigma_k T_M}{1 - e^{-\sigma_k T_M}} \tag{33}$$

$$\alpha_k = \arccos(y_k / A_k) \tag{34}$$

$$\beta_k = \arcsin(y_{k+m}/A_k) \tag{35}$$

The phase is defined from expression (17):

$$\phi_k = -\tan^{-1}(b_k/a_k) \tag{36}$$

Its average estimate is simply

$$\phi_k \approx (\alpha_k + \beta_k)/2 \tag{37}$$

The amplitudes and phases can be identified for a given component as just shown. Consequently, the eigenvectors W_k can be estimated as well by using the time histories of all the degrees of freedom of the system under study. The coupling between any two aeroelastic degrees of freedom can be studied using the discrete cross correlation in the time domain prior to performing the analysis.

Validation of the Present Approach

The present approach, which uses a least-squares solver on a moving block, will be called the least-squares moving-block technique (LSMBT). The classical moving-block technique is denoted by MBT. The LSMBT will be applied to a number of selected analytical test cases to evaluate the present method. For comparison, the results from some of the same test cases using the MBT will be included. Then, LSMBT will be used to find the frequency and damping of the pitch/flap/lag modes from results of a given helicopter simulation (REXHEL).[10] Time histories will be generated from exact analytical expressions, which take the following two forms:

$$x(t) = \sum_{k=1}^{m} A_k e^{-\sigma_k t} \cos(2\pi f_k t + \phi_k) \tag{38}$$

for a constant coefficient system and

$$x(t) = \sum_{k=1}^{m} F_k(t) e^{-\sigma_k t} \cos(2\pi f_k t + \phi_k) \tag{39}$$

$$F_k(t) = A_k[1 + \epsilon_1 \cos(2\pi f_0 t + \gamma_1) + \epsilon_2 \cos(4\pi f_0 t + \gamma_2) + \cdots] \tag{40}$$

for a periodic system. As follows, analytical test cases 1, 2, and 4 make use of analytical expression (38), and test case 3 makes use of Eqs. (39) and (40). The time increment and frequency resolution used for the four analytical cases are given as

$$\Delta t = 0.01 \text{ s} \qquad \Delta f = 0.5 \text{ Hz}$$

Analytical Test Case 1

This example treats the case of two modes with close frequencies. The time signal and corresponding Fourier spectrum are respectively shown in Figs. 1 and 2. The dominant frequencies are visualized in Fig. 2; their calculated values match the exact analytical values shown in Table 1. The damping estimates using the LSMBT and MBT together with the exact analytical values of damping are given in Table 1. The LSMBT estimates the damping for both modes (σ_k^1, σ_k^2) with block lengths $N_1 = 1/(f_1 \Delta t)$ and $N_2 = 1/(f_2 \Delta t)$. The estimated values of damping given by the LSMBT are clearly in better agreement with the exact values than the estimates given by the MBT. The MBT damping estimates are off because of leakage resulting from frequency closeness; naturally, the lower damping will dominate the results in most applications. The limitation in accuracy in case of two close frequencies for the MBT is removed by the LSMBT. The extreme case of two identical frequencies with two different dampings can be solved using

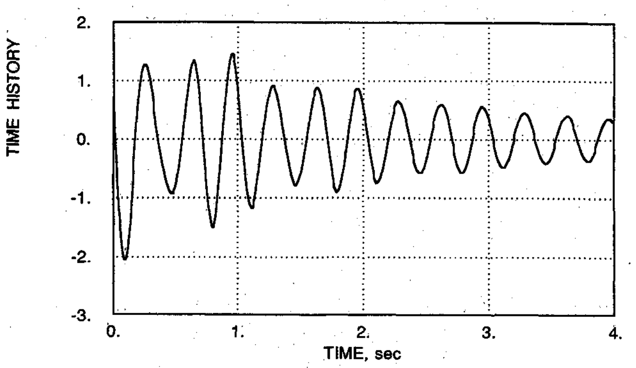

Fig. 1 Transient time history for the case of two close frequencies.

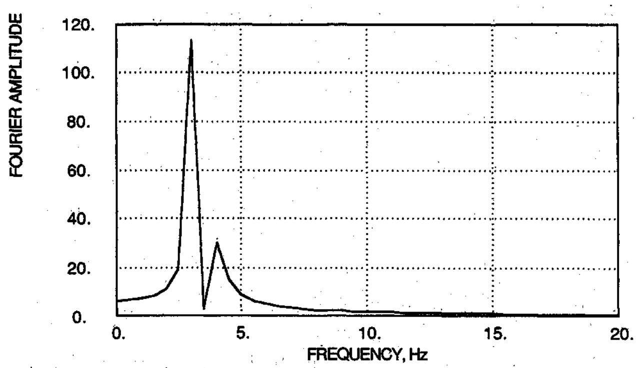

Fig. 2 Fourier spectrum for the case of two close frequencies.

Table 1 Frequency and damping results (Hz) for analytical test case 1

	LSMBT		MBT		
Mode number	Damping (N1)	Damping (N2)	Damping	Damping (exact)	Frequency (exact)
1	0.444	0.457	0.319	0.400	3.0
2	1.202	1.338	0.368	1.200	4.0

the LSMBT if information from the time derivative of the time signal is included in the analysis. However, the damping estimates will then be given as the solution of a set of nonlinear algebraic equations.

Analytical Test Case 2

In this case, the LSMBT will be used solely to estimate the eigenvalues as well as the eigenvector components from a multimode transient signal. Four modes are considered in this example. The time signal is depicted in Fig. 3. The associated Fourier spectrum (Fig. 4) clearly shows the four dominant frequencies. The frequency and damping results are given in Table 2. The application of the MBT in this case gives good results for dampings corresponding to low frequencies and particularly bad estimates for the dampings associated with higher frequencies. The relative error using MBT is as high as 88% compared to a maximum of 24% for the LSMBT. Clearly, the LSMBT gives better results in case of a multimode time signal. The amplitude and phase of the eigenvector components are estimated in the first moving block of maximum length. The results are shown in Table 3; the agreement with the exact values is good. As stated earlier, the advantages of the LSMBT over the MBT is the capability to estimate the eigenvector components.

Analytical Test Case 3

The analysis of aeroelastic stability from the time history data of the transient dynamic response is significantly influenced by the rotor periodicity especially at high values of the advance ratio μ.

For a typical pitch/flap/lag aeroelastic motion with frequencies f_1, f_2, f_3 and rotor frequency f_0, the following frequencies might appear in the frequency spectrum

$$f_{nk} = |f_k \pm nf_0| \qquad k = 1,2,3 \qquad n = 0,1,2,3,\ldots \tag{41}$$

The problem of identification becomes more elaborate; the Fourier amplitudes and dampings also need to be considered in the analysis for the proper identification of the dynamic characteristics. At any rate, aeroelastic roots are more easily identified from hover $\mu = 0$; then they can be continuously tracked for higher values of the advance ratio μ. As an illustration, the case of two modes with periodic amplitudes is considered here. Only the first harmonic in Fourier expansion [Eq. (40)] will be retained for the data generation. The rotor frequency f_0 is given the value of 1.5 Hz with $f_1 = 2.5$ Hz and $f_2 = 5$ Hz. Using the preceding formula, the following frequencies, in Hertz, are expected to show in the frequency spectrum:

1, 2.5, 3.5, 4, 5, 6.5

Table 2 Frequency and damping results (Hz) for analytical test case 2

	LSMBT		MBT			
Mode number	Damping	Damping error, %	Damping	Damping error, %	Damping (exact)	Frequency (exact)
1	0.625	4	0.597	0	0.600	1.0
2	0.495	24	0.437	9	0.400	3.0
3	2.420	21	0.250	88	2.000	5.0
4	3.852	10	0.699	80	3.500	8.5

Table 3 Amplitude and phase results for analytical test case 2

Mode number	Amplitude (LSMBT)	Amplitude (exact)	Amplitude error, %	Phase (LSMBT)	Phase (exact)	Phase error, %
1	2.438	2.400	2	1.570	1.560	1
2	1.702	1.700	0	0.853	0.850	0
3	1.282	1.200	7	2.391	2.420	1
4	0.968	0.830	16	0.001	0.000	0

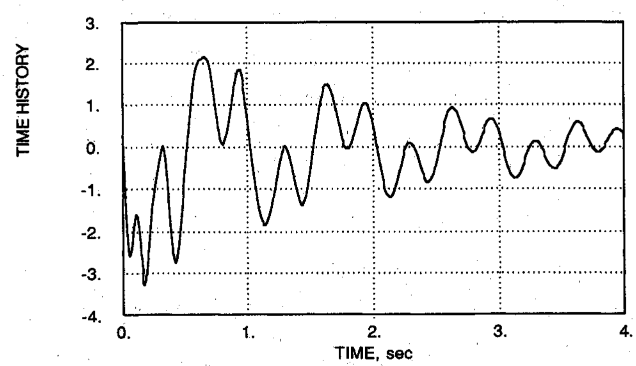

Fig. 3 Transient time history for the case of multiple active modes.

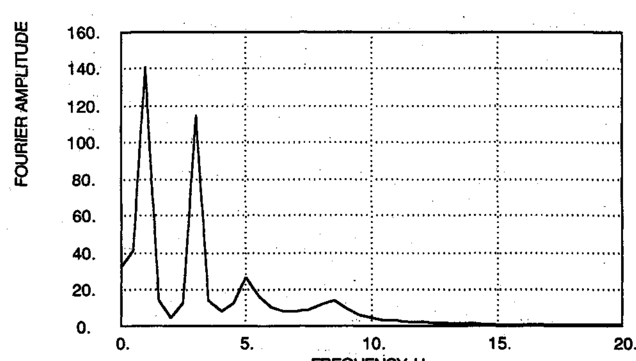

Fig. 4 Fourier spectrum for the case of multiple active modes.

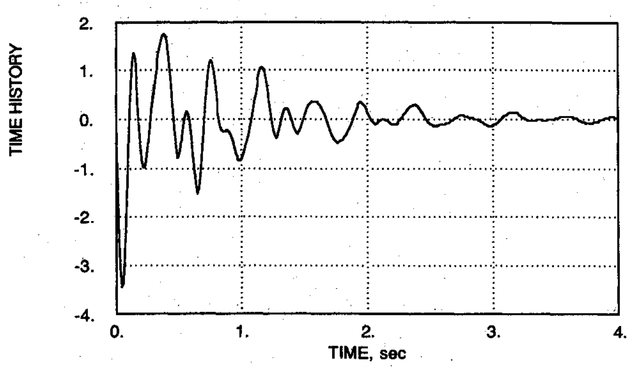

Fig. 5 Transient time history for the case of periodic modes.

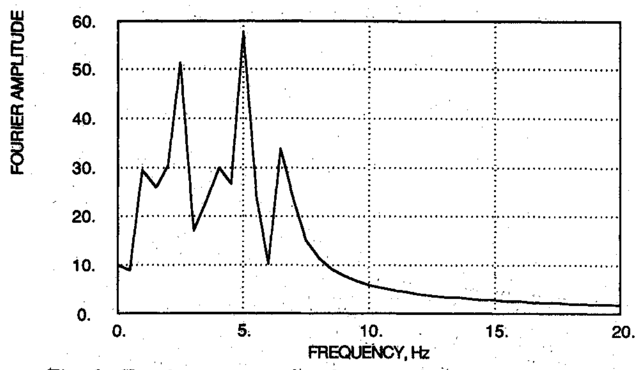

Fig. 6 Fourier spectrum for the case of periodic modes.

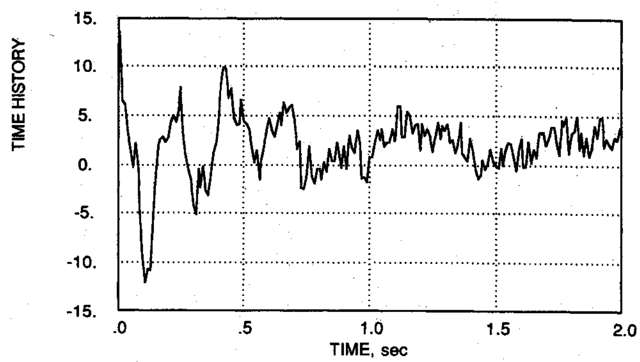

Fig. 7 Transient time history for the case of multiple modes with 30% noise level.

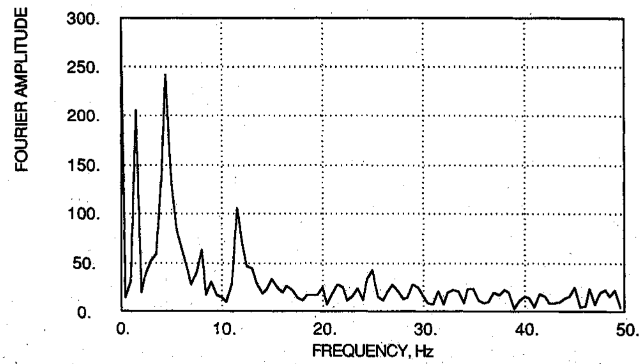

Fig. 8 Fourier spectrum for the case of multiple modes with 30% noise level.

Figures 5 and 6 depict, respectively, the time signal and its corresponding Fourier spectrum. All of the preceding frequencies are present in the Fourier spectrum except the 3.5-Hz frequency, which merged with the 4-Hz frequency. The frequency and damping results using the LSMBT are displayed in Table 4. The accuracy obtained is good.

The amplitude and phase coefficients contained in the Fourier expansion [Eq. (40)] can be estimated similarly to the constant coefficient case by substitution of Eq. (40) into Eq. (39). It can be shown that the Eq. (39) then takes the following general form:

$$x(t) = \sum_{n,k} A_{nk} e^{-\sigma_k t} \cos(2\pi f_{nk} t + \phi_{nk}) \tag{42}$$

Analytical Test Case 4

The analysis of test data to extract eigenresult information is an important application of parameter identification techniques. This test case is concerned with the effect of noise on the prediction of frequency and damping. Consider a typical rotor blade with three active modes:

Mode 1 (first lag):

$$\sigma_1 = 0.56 \text{ Hz} \quad f_1 = 1.50 \text{ Hz}$$

Mode 2 (first flap):

$$\sigma_2 = 2.20 \text{ Hz} \quad f_2 = 4.50 \text{ Hz}$$

Mode 3 (second flap):

$$\sigma_3 = 1.74 \text{ Hz} \quad f_3 = 11.50 \text{ Hz}$$

The time signal with noise x_R is written as a sum of analytical expression (38) and of a random function of time, rand, which takes values between -1 and 1

$$x_R(t) = x(t) + \frac{c\,|x_0|}{100}\,\text{rand}(t) \tag{43}$$

where constant c is given in percent of noise level. Figures 7 and 8 describe the time signal and Fourier spectrum for a 30% noise level. The three dominant frequencies are easily recovered using the fast Fourier transform up to a 60% noise level. The values of the damping estimates are shown in Table 5 together with the exact values for comparison. The performance of the present approach for this test case is depicted in Fig. 9, which displays the relative damping error ($|\sigma - \sigma_{\text{exact}}|/\sigma_{\text{exact}}$) vs noise level. For low noise levels up to 25%, the accuracy is about 10%. For a 40% noise level, the accuracy is about half as much. The results demonstrate that this method performs quite well in the presence of noise and can certainly be used to identify aeroelastic modes from measurements of time history.

Application to X-Wing Aeroelastic Modes

The time response to an initial disturbance (at a given location on the X-wing) is computed using REXHEL[10] for the flap, lag, and pitch motions. The three time histories are then analyzed using the LSMBT. The collected information from the analysis of the flap, lag, and pitch time signals is used to

Table 4 Frequency and damping results (Hz) for analytical test case 3

Mode number	Frequency (LSMBT)	Frequency (exact)	Damping (LSMBT)	Damping (exact)	Damping error, %
1	1.0	1.0	1.077	0.900	17
2	2.5	2.5	0.945	0.900	5
3	4.0	4.0/3.5	0.945	—	—
4	5.0	5.0	1.855	1.700	9
5	6.5	6.5	1.747	1.700	3

Table 5 Damping results (Hz) for analytical test case 4

Noise level	0%	15%	30%	45%	60%	Exact
First lag	0.645	0.573	0.496	0.417	0.339	0.560
First flap	2.124	2.051	1.908	1.758	1.622	2.200
Second flap	1.748	1.862	1.927	1.877	1.710	1.740

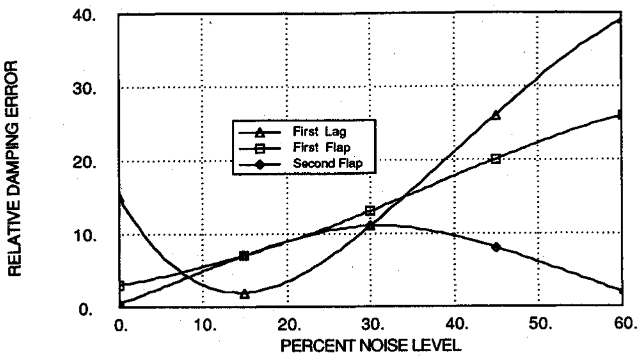

Fig. 9 Relative damping error vs percent noise level.

Table 6 Frequency and damping results (Hz) for the X-Wing aeroelastic modes

Mode	Natural frequency	Frequency (LSMBT)	Damping (LSMBT)	Damping ratio, %
Flap	5.70	5.0	1.604	31
Lag	6.15	6.5	1.300	20
Pitch	39.80	36.5	2.830	8

estimate useful engineering values for the damping of the different motions. The frequency and damping results, together with the natural frequencies, are displayed in Table 6. In this application, it was found that the lag motion was weakly coupled with the other aeroelastic motions. Similarly, the flap motion was weakly coupled with the pitch motion. Nonetheless, the results show a good correlation among the frequencies and dampings. The advance ratio and rotor frequency in this case are

$$\mu = 0.68, \qquad f_0 = 2.6 \text{ Hz}$$

The time increment and frequency resolution used in this application are

$$\Delta t = 0.0125 \text{ s} \qquad \Delta f = 0.5 \text{ Hz}$$

the block length with respect to the pitch frequency (Table 6) is then

$$N_P = \frac{1}{0.0125 \times 36.5} = 2$$

The MBT cannot be used for only two points. However, even in this case, the LSMBT is able to estimate the damping using a larger block length from a lower dominant frequency.

Conclusion

The least-squares moving-block technique (LSMBT) has been shown to give better estimates than the moving-block technique for the dampings in four analytical test cases. The most difficult parameters to estimate are the dampings; this difficulty is intuitively expected because of the nonlinearity the damping parameters introduce in the estimation problem. A powerful way to estimate dominant frequencies (modes) is by a search of local maxima in the frequency domain whereas dampings are better estimated in the time domain as demonstrated by this new method. The additional advantage of the LSMBT over the classical approach is the capability to estimate eigenvector components with good accuracy. This feature is useful in assessing the coupling of the different modes. The approach presented in this paper can also be used to perform periodic modal identification, which is of significant importance for rotary wings flying at moderate to high speeds. Finally, this method can be applied to extract modal information from experimental data because of the good confidence shown in the results obtained in the presence of noise.

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